Dr. Marques Sophie
Office 519

Algebra 1

Fall Semester 2014

marques@cims.nyu.edu

Quiz #4

Problems:

1. (20 pt) Let $\phi : D_n \to \mathbb{Z}/2\mathbb{Z}$ be the map given by

$$\phi(x) = \begin{cases} 0 & \text{if } x \text{ is a rotation} \\ 1 & \text{if } x \text{ is a reflection} \end{cases}$$

   (a) Show ϕ is a homomorphism. (Hint: Remember it is enough for this to consider the product of two reflection, the product of a reflection and a rotation, the product of two rotations.)

   (b) What is $ker(p)$? What is $Im(p)$?

   Solutions:

   (a) The product of two reflections is a rotation around the intersection point of the two reflection axes; the product of a reflection and a rotation is a reflection; and the product of two rotations is again a rotation.

   (b) $ker(p) \simeq \mathbb{Z}/n\mathbb{Z}$ *is the cyclic subgroup generated by a rotation through* $360/n$ *degrees.*
   $Im(p) = \mathbb{Z}/2\mathbb{Z}$.

2. (20pt) Consider the group $G = S_3 \times \mathbb{Z}/6\mathbb{Z}$.

   (a) Determine the set of orders of elements in G, that is, the set $\{|g||g \in G\}$.

   (b) Prove that G is not cyclic.

   Solutions:

   (a) Orders of elements in S_3*:* 1, 2, 3*; Orders of elements in* $\mathbb{Z}/6\mathbb{Z}$*:* 1, 2, 3, 6*; Orders of elements in* $S_3 \times \mathbb{Z}/6 : \mathbb{Z}$*;* 1, 2, 3, 6.

   (b) The order of G *is* 36*, but there are no elements of order 36 in G. Hence G is not cyclic.*

3. (10 pt) List the group of order 6 without proof, up to isomorphism.

   Solutions:
   $\mathbb{Z}/6\mathbb{Z}$, D_3.